Introduction to Modern Macro

Honours Intermediate Macro

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Motivation on the "Modern Macro" Approach

This course will concentrate on theory rather than data (which is extremely important, but will be covered in other classes). "Modern" here is synonymous with how macro is taught at the graduate, rather than typical undergraduate courses.

Why Model using Precise Math?

If you can describe a concept in words, you can write it down in math. With math, you can follow through mechanically, with little room for error, and ensuring self-consistency. **"Math is so that you don't have to think."**

Using precise mathematical language will (1) uncover unanticipated consequences implicit in your assumptions; (2) keep everyone honest; (3) provide a framework to investigate changes in assumptions; (4) allow you to add reality, nest models, and do quantitative analysis.

The mathematics should be no fancier than is necessary (but the more "reality" you want to put into a model, the more math is required).

Why Not Just use Statistics?

Purely statistical analysis of data is very useful, but without some background model, the statistics and regression coefficients are difficult to interpret. Moreover, if you only use historical data, you are unable to predict responses to changes in government policy and perform counterfactuals (i.e., called the "Lucas Critique"). In essence, without some underlying model in the back of your mind, you are unable to understand the incentives which might explain patterns in the data.

Furthermore, academic economists are not in the business of "forecasting". In fact, macro theory says that it is not possible to systematically forecast asset markets, etc.

Why Start with Models too Simple to Reflect Reality?

You want to walk before you can run: **"All models are wrong, but some models are useful"**. Simple models will enable you to build familiarity with the tools, and understand concepts in isolation. Clean intuition always comes from the simplest model, leaving in only enough reality to understand your questions.

Warmup: Asset Pricing and PDV

- We will first build some general tools, apply them to asset pricing, then keep using the tools throughout the course for other examples.
- Time is discrete: $t = 0, 1, ...$
- Asset has deterministic payoffs: $y_t \geq 0$ for $t = 0, 1, ...$ (i.e., forever)
  - Does the *infinite horizon* matter? Is it a useful approximation?
- Discount the future: $0 < \beta < 1$

For example, 1-period discount rate $r > 0 \Rightarrow \beta = \dfrac{1}{1+r}$. Note that r, β are related to period length, e.g. 1 quarter.

What is the value of this asset?

$$P = \sum_{t=0}^{\infty} \beta^t y_t = y_0 + \beta y_1 + \beta^2 y_2 + ... \quad \text{(Present Discounted Value)}$$

For example, if $y_t = \bar{y}$, constant:

$$P = \sum_{t=0}^{\infty} \beta^t \bar{y} = \bar{y} \sum_{t=0}^{\infty} \beta^t = \bar{y}\left(\frac{1}{1-\beta}\right) = \left(\frac{1+r}{r}\right)\bar{y}$$

Geometric Series Expressions

$$\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$$

$$\sum_{t=0}^{\tau} \beta^t = \frac{1-\beta^{\tau+1}}{1-\beta}$$

$$\sum_{t=\tau}^{\infty} \beta^t = \sum_{t=0}^{\infty} \beta^t - \sum_{t=0}^{\tau-1} \beta^t = \frac{1}{1-\beta} - \frac{1-\beta^{\tau}}{1-\beta} = \frac{\beta^{\tau}}{1-\beta}$$

Does this always work? $|\beta| < 1$?

Useful tool: Ask an LLM (e.g., ChatGPT, Claude) to verify these formulas:

- "What is the sum of b^t from t=0 to infinity?"
- "What is the sum of b^t from t=0 to tau?"

Asset Pricing Example 1

$$y_t = \lambda^t$$

$$P = \sum_{t=0}^{\infty} \beta^t \lambda^t = \sum_{t=0}^{\infty} (\lambda\beta)^t = \frac{1}{1-\lambda\beta} \quad \text{if } |\lambda\beta| < 1$$

That is, dividends can't grow faster than discount rate.

Asset Pricing Example 2

$$y_t = \begin{cases} \bar{y} & \text{for } t = 0, \dots, T \\ 0 & \text{for } t > T \end{cases}$$

$$P = \sum_{t=0}^{\infty} \beta^t y_t = \bar{y} \sum_{t=0}^{T} \beta^t + 0 \cdot \sum_{t=T+1}^{\infty} \beta^t = \bar{y} \cdot \frac{1-\beta^{T+1}}{1-\beta}$$

Recursive Formula for Asset Pricing

$$P_t = \sum_{j=0}^{\infty} \beta^j y_{t+j} \quad \text{at time } t \text{ (Sequential formulation)}$$

$$P_t = y_t + \beta \underbrace{\left(\sum_{j=0}^{\infty} \beta^j y_{t+j+1} \right)}_{P_{t+1} \text{ by definition}} \quad \text{i.e., factor out a } \beta$$

$$\Rightarrow P_t = y_t + \beta P_{t+1}$$

Value today = dividend today + discounted value tomorrow.

- Since it has "t" and "$t+1$": a *difference equation*
- Linear difference equation (since no terms like P_t^2 or $P_t \cdot P_{t+1}$)
- First-order since only "t" and "$t+1$" (i.e., no "$t+2$")
- A discretization of a first order linear differential equation

Some General Concepts in Macro

Relating some of the key concepts to our simple asset pricing example:

Endogenous vs. Exogenous

Exogenous means given from outside. Examples are: (1) an equation for the evolution of money supply, independent of any components of the economic model; (2) a process describing how the salary of an individual would evolve—without any ability for the worker to affect it.

Endogenous means within the system/model. Examples are: (1) prices of assets determined by the trades and values of agents; (2) supply of a good determined by the production decisions of firms; and (3) the salary of an individual if they are able to change it by choosing their investment in education.

Some Guiding Principles for Our Models

In general, we want to maintain the following in our models:

- **People:** We want an economics of people—and institutions that represent them such as firms and governments—rather than a set of disconnected equations representing the economy. Among other reasons, this is necessary to think about individuals to analyze topics such as income inequality or responses to government policy.
- **Dynamic:** The idiosyncratic (i.e. particular to an individual) and aggregate (i.e., common across individuals) state in the economy evolves over time.
  - e.g. use *difference equations*
- **Expectations:** People form expectations over what might happen in the future. Past observations are part of how they make decisions, but they also take into account the dynamic evolution of the economy, and the internal "model" of how they think the world operates.
  - e.g. forecasts of time $t+1$ made at time t. Mathematical expectation: $\mathbb{E}_t\left[x_{t+1}\right]$
- **Agent Choices and Incentives:** Where possible, derive the evolution of the economy from the choices of individuals. An agent making a choice needs to take into account dynamics as well as her expectation of future events.
  - e.g. solve a maximization problem

- **Equilibrium Conditions:** Interpreted broadly, the idea is that prices should reflect the decisions of individuals rather than being taken as given. When people say "General Equilibrium" they just mean that there are no exogenously given prices.
  - e.g. solve systems of equations to find prices that "clear markets" based on everyone's decisions

Other Useful Concepts

- **Stochastic:** A deterministic evolution of the state is a good place to start, but randomness is often more realistic. This makes dynamics and expectations even more complicated.
  - e.g. will use *stochastic difference equations* and *Markov processes*.
  - e.g. $\mathbb{E}_t\left[x_{t+1}\right] \equiv \mathbb{E}\left[x_{t+1} \mid x_t\right]$, an expectation of tomorrow conditional on information available today. Model determines information sets.
- **Recursive:** Sometimes dynamics are easiest solved by writing everything recursively (today is written in terms of the state today and the forecasts of tomorrow, where the same formula could apply tomorrow)
  - e.g., the value of a stock today is the dividend today + the price you could sell it tomorrow. Use same formula when tomorrow comes, then solve a functional equation.
- **Aggregated:** It is easiest to start with every individual looking the same, but making their own decisions. From this, you can usually "aggregate" so that summary statistics of the economy evolve from what looks like a single agent (often called a "representative agent"). This is a result, not an assumption!
- **Heterogeneous:** If you are interested in things like income inequality or differences between the productivity of firms, you will need to take into account a whole bunch of "heterogeneous" agents interacting in the economy.
- **Linear:** Making some crude assumptions on linearity of processes and functions will greatly simplify the algebra, and capture most of the intuition. Useful, but not necessary.

The difficulty of macro is that you usually need to take into account all of these at the same time. Basically: Take micro models to represent an agent, make it more dynamic, add in complicated expectations over the future, have a huge number of these individuals interacting, and then have prices reflected by decisions of everyone. Because this is so complicated, we will start with very simple models to gain intuition, and then build up more realism.

Rationality and Arbitrage

There is a great deal of irrationality in human behavior, but to what extent does it matter for macroeconomic decisions? Is it systematically biased (i.e., irrationality and mistakes don't cancel out with large numbers of people)?

The empirical reason we don't focus on ad-hoc behavioral assumptions is arbitrage. If a large number of people are making systematically bad decisions, then in the macro-economy with asset markets, you can pump a huge amount of money out of them. But we don't really see this as people either don't participate or adapt. It doesn't take the whole population to be "rational", just enough deep pocketed investors to move markets. Asset prices reveal information. Principles to consider:

> **"You can fool some of the people some of the time, but not all of the people all of the time."**
>
> **"If you are so smart, why aren't you rich?"**

That said, carefully limiting the degree of rationality—typically called "bounded rationality" in macro—is important, but we need to start with rationality and full information as a baseline.