ECON307 Problem Set 3

Jesse Perla

Question 1

Consider two scenarios for a consumer planning consumption with income process $y_t = y_0\delta^t$ for all $t \geq 0$ and $F_0 = 0$.

Scenario 1 for Consumer: The consumer maximizes the following welfare

$$
\begin{aligned}
U \equiv \max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \log(c_t) \\
\text{s.t.}\ & F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \\
& \text{(transversality condition)}
\end{aligned}
$$

Scenario 2 for Consumer: The consumer faces the same problem as Scenario 1, except with **no borrowing**: $F_{t+1} \geq 0$ for all $t \geq 0$, and the initial level of y_0 is potentially different (define it as y_0^{NB}).

Define the PDV of utility (i.e., the welfare) of this as U^{NB}.

Question 1.1

Let $\beta = 0.95$, $R = 1.04$, $\delta = 1.02$, $y_0 = 1$, and $y_0^{NB} = 1$. Calculate U and U^{NB}.

Question 1.2

Let $y_0 = 1$. Now find a y_0^{NB} such that $U = U^{NB}$. The difference between y_0 and y_0^{NB} is the amount of sacrifice in terms a consumer with a borrowing constraint would pay to be free to borrow. A measure of the welfare loss of the no borrowing constraint.

Question 1.3

Maintain $y_0 = 1$. Now, let $\beta = 0.99$, $R = 1.04$, and $\delta = 1.01$. What is c_0 and F_1 here under Scenario 1? Repeat part (b) to find y_0^{NB} such that $U = U^{NB}$ with these new parameters. What can you conclude about the welfare cost of no borrowing in this case?

Question 2

Let the consumer have power utility,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 1$$

Given $F_0 = 0$, $B \geq 0$, $\beta R = 1$, and the deterministic income stream $y_t = \delta^t$, the consumer maximizes

$$\begin{aligned}
\max_{\{c_t, F_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t. } & F_{t+1} = R(F_t + y_t - c_t), \quad \forall t \geq 0 \\
& F_{t+1} \geq -B \\
& F_0 = 0 \\
& \text{(transversality condition)}
\end{aligned}$$

Question 2.1

Derive the Euler equation as an inequality, and the condition for it holding with equality.

Question 2.2

Let $\delta > 1$ and $B = \infty$. What is $\{c_t\}_{t=0}^{\infty}$?

Question 2.3

Let $\delta > 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

Question 2.4

Let $\delta < 1$ and $B = 0$. What is $\{c_t\}_{t=0}^{\infty}$?

Question 2.5

Assume that the consumer optimally eats their entire income each period, i.e., $c_t = y_t = \delta^t$ which implies $c_{t+1} = \delta c_t$. Setup, using dynamic programming, an equation to find the value $V(c)$ recursively.

Question 2.6

Guess that $V(c) = k_0 + k_1 c^{1-\gamma}$ for some undetermined k_0 and k_1. (Note: this equation deliberately avoids any t subscripts, making it a truly recursive expression.) Solve for k_0 and k_1 and evaluate $V(1)$ (i.e., the value of starting with $c_0 = 1$).

Question 3

Consider a Markov chain with two states: U for unemployment and E for employment.

- With probability $\lambda \in (0, 1)$, a person unemployed today becomes employed tomorrow.
- With probability $\alpha \in (0, 1)$, a person employed today becomes unemployed tomorrow.

Question 3.1

Let $N \geq 1$ be the number of periods until a currently **unemployed** person becomes **employed**. Calculate $\mathbb{E}[N]$.

Question 3.2

Let $M \geq 1$ be the number of periods until a currently **employed** person becomes **unemployed**. Calculate $\mathbb{E}[M]$.

Question 3.3

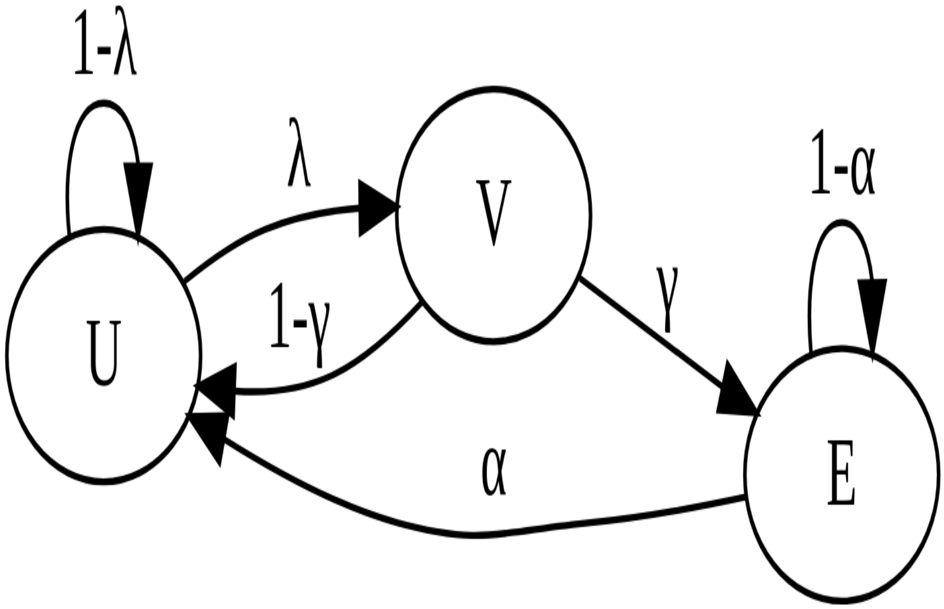
Please compute the fraction of time an infinitely lived person can expect to be **unemployed** and the fraction of time they can expect to be **employed**.

Question 4

An economy has 3 states for workers:

- U: unemployment
- V: verification (found a potential employer and being verified for fit)
- E: employed (verified and working)

The probabilities of jumping between states each period are shown below:



i.e., probability γ they are a good fit, and the verification takes 1 period.

Question 4.1

Write a Markov transition matrix for this process, P.

Question 4.2

Write an expression for the stationary distribution across states in the economy, $\pi \in \mathbb{R}^3$ (You can leave in terms of P).

Question 4.3

If a worker is U today, write an expression for the probability they will be employed exactly j periods in the future (considering any possible transitions which end in employment at j periods).

Note: This is only looking at j periods into the future, i.e., this is ***not*** *the probability that they become employed at least once during the j periods, which is a much more difficult calculation.*

Question 4.4

Assume that $\alpha = 0$, $\lambda = 0$. Is the stationary distribution unique? If not, describe the sorts of distributions that could exist and the intuition from the perspective of the Markov chain.